White level

1. **(5 credits)** Thumbelina asked old Mr. Mole how old he was. Mr. Mole replied: “If you multiply my age by 3, and then subtract 16, you will get 17”. How old was Mr. Mole?

   Answer: 11
   Solution: We make the equation 3x-16=17, its solution is 11.

2. **(5 credits)** When Robinson Crusoe looked at his calendar, he thought, 'In 363 days the number of the day will be 14 times greater than today.' In what month and on what day did Robinson Crusoe look at the calendar?

   Answer: on the 2nd of March
   Solution: After 365 or 366 days (depending on whether it was a leap year or not) the same date comes. According to the problem statement, 2 or 3 days before the mentioned event the number of the day was 14 times greater, i.e. the day mentioned by Robinson Crusoe was the 14th or 28th day of the month accordingly. But 2 or 3 days after the 14th day the 16th or 17th day comes, and this means that this option is not suitable. 2 or 3 days after the 28th day the 30th, 31st or 1st day usually comes, and this means that this option is also not suitable. The only exception is the day which comes 2 days after February 28th, i.e. March 2nd. (Note: exactly 363 days always pass between March 2nd and February 28th of the next year, because the added day, February 29th, does not fall within this interval.)

3. **(7 credits)** Professor Dumbledore eats the same number of lemon drops every day. Lemon drops are sold in large, medium or small packages. The large package contains three times more lemon drops than the small one, and the medium package contains two times more lemon drops than the small one. All lemon drops in the big package will be eaten by Dumbledore in exactly 50 days. How many days does Dumbledore need to eat all lemon drops in the medium package?

   Answer: 33
   Solution: Let the small package contain n lemon drops. This means that the medium package contains 2n lemon drops, and the large package contains 3n lemon drops. According to the problem statement, 3n lemon drops will be eaten by Dumbledore in 50 days, i.e. Dumbledore eats 3n/50 lemon drops in a day. Therefore, the medium package containing 2n lemon drops will be eaten in 2n / (3n/50) days, i.e. 100/3 = 33 ⅓ days. Therefore, such quantity of lemon drops will be enough for 33 days (but not for 34 days). Note: an integer number of days must be specified in the answer anyway.

4. **(8 credits)** Bill Weasley was 30 years old when his daughter Victoire was born. How old is Bill in 2020, if he was six times her age on her birthday in 2006?

   Answer: 50 or 51
   Solution: The age difference between Bill and his daughter on her birthday is the same as at the time of her birth: 30 years. In 2006, this difference was five times the daughter’s age, which means that his daughter was 6 years old, i.e. she was born in 2000, and Bill was born 30 years earlier, i.e. in 1970 or in 1969. Therefore, he is 50 or 51 years old in 2020.

5. **(10 credits)**The Brave Little Tailor cut the square piece of fabric into 7 parts along three straight lines that do not go through the vertices of the square and got three pentagons. How many corners do the remaining four figures have?

   Answer: 13
   Solution: Three lines intersect at three points inside the square (otherwise the number of the parts would be less than seven) and intersect the sides of the square at six other points. Each of the interior points is the vertex of four figures (12 corners in total), each of the points of intersection with the sides is the vertex of two figures (12 corners in total), and the corners of the square are another four corners of the figures. Therefore, the total number of the corners is 28. After subtracting five corners in each pentagon from the total number of the corners, we get 28-3*5=13 corners.

6. **(10 credits)** Two vertices of a square lie on the abscissa axis, and the other two vertices lie on the parabola y = x2. What is the area of this square?

Answer: 16
Solution: If one of the vertices of the square on the abscissa axis has the abscissa a, the abscissa of the second vertice is -a. The other two vertices have the same abscissa, and their ordinates are a^2. We equate the lengths of the horizontal and vertical sides and get the equation $a^2 = 2a$, from which a = 2, i.e. the side of the square is 4 and its area is 16.

7. **(12 credits)** Edward Scissorhands and his brother cut two identical rectangles into parts. Edward got two rectangles with a perimeter of 8 cm, and his brother got two rectangles with a perimeter of 13 cm. What perimeter did the original rectangles have?

Answer: 14
Solution: Since Edward and his brother got rectangles, they cut along straight lines parallel to the sides of the original rectangles. Since the rectangles were identical, the cut line passed through the midpoints of opposite sides. Therefore, if we designate the sides of the original rectangle as 2x and 2y, then Edward got rectangles with sides 2x and y, and his brother got rectangles with sides x and 2y, and we get the following equations for the perimeters: 2 (2x + y) = 8 and 2 (x + 2y) = 13. Actually we can just solve them and find the values of x and y and the perimeter 4 (x + y). However, there is a more elegant and faster way to find the answer: we can sum up both equations and get 6x + 6y = 21, from which x+y = 3.5 and 4(x+y)=14.

8. **(13 credits)** In the division sentence AB / CD = E, different letters represent different digits appearing in descending order. Find the dividend.

Answer: 86
Solution: After replacing the letters with digits we get 86/43 = 2. Let us prove that this option is the only possible one. The quotient cannot be 3 or exceed 3, since in this case the divisor must be at least 54, and the product of 3 and 54 is a three-digit number. The quotient also cannot be equal to 1, because the dividend is not equal to the divisor. Therefore, the quotient is 2. But in this case the divisor is at least 43 and at most 49, since the dividend is less than 100. Therefore, C = 4. But since E<D<C according to the problem statement, then 2<D <4, from which D = 3. After multiplying 2 by 43, we get the two-digit number AB=86.

9. **(15 credits)** A pair of natural numbers (a, b) satisfies the equation ab + a + b = 2020. Find all possible values of the sum a+b (list them separated by commas in your answer).

Answer: 88
Solution: After adding 1 to both parts, we get the product (a + 1) (b + 1) on the left, and 2021 = 43 * 47 on the right. Numbers 43 and 47 are prime (they have no other divisors except 1 and themselves). Since the numbers a + 1 and b + 1 are not less than 2, then one of them is 43, and the other is 47, so their sum is (43-1)+(47-1)=90-2=88.

10. **(15 credits)** There are n knights on the chessboard. Among any 8 knights, there is always at least a pair of knights that can beat each other. What is the largest possible n?

Answer: 14
Solution: Let's consider the option with 14 knights first. For this purpose, we place knights on all squares of the first rank, except for the rightmost square, as well as on all squares of the third rank, except for the leftmost one. There are 7 pairs of knights on the chessboard. For each knight on the first rank there is a pair on the third rank: a knight, standing one square to the right; these two knights can beat each other. Whatever 8 knights you take, there are two paired knights among them. Now let's make sure that there is no appropriate combination for any larger number of knights. If we have 15 knights, 8 of them are on cells of the same color (if the standard checkerboard coloring is used). None of these eight knights beats any other knight of this set.